CS 2209 Assignment 1

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* 1. prove that p → (q→r) and (p→q)→r is not equivalent

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cond Identity  Cond Identity  Associative  DeMorgans | p → (q→r)  ¬ p∨(q→r)  ¬ p∨(¬q∨r)  (¬p∨¬q)∨r  ¬(p∧q)∨r |  | (p→q)→r  ¬(p→q)∨r  ¬(¬p∨q)∨r  ¬¬p∧¬q∨r  p∧¬q∨r | Cond Identity  Cond Identity  DeMorgans  Double Negation |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P | Q | R | ¬p | ¬q | (p∨q) | ¬(p∧q) | ¬(p∧q)∨r | p∧¬q | p∧¬q∨r |
| T | T | T | F | F | T | F | T | F | T |
| T | T | F | F | F | T | F | F | F | F |
| T | F | T | F | T | T | F | T | T | T |
| T | F | F | F | T | T | F | F | t | T |
| F | T | T | T | F | T | F | T | F | T |
| F | T | F | T | F | T | F | F | F | F |
| F | F | T | T | T | F | T | T | F | T |
| f | f | f | t | t | f | t | t | f | f |

The two columns are not the exact same and as such the statements are not equivalents

Not equivalent when p=q=r=F

* 1. (p∧q)∨(¬p∨¬q)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| P | Q | ¬p | ¬q | (P∧q) | (¬p∨¬q) | (p∧q)∨(¬p∨¬q) |
| T | T | F | F | T | F | T |
| T | F | F | T | F | T | T |
| F | T | T | F | F | T | T |
| f | f | t | t | f | t | T |

The truth table shows it is always true therefore it is a tautology

* 1. (p→¬q)→(p→q)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| P | Q | ¬q | p→¬q | p→q | (p→¬q)→(p→q) |
| T | T | F | F | T | T |
| T | F | T | T | F | F |
| F | T | F | T | T | T |
| f | f | t | t | t | T |

The truth table shows that is it neither a tautology or a contradiction as there is both true and false values

* 1. (p→q)→(¬q→¬p)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| P | Q | ¬p | ¬q | p→q | ¬q→¬p | (p→q)→(¬q→¬p) |
| T | T | F | F | T | T | T |
| T | F | F | T | F | f | T |
| F | T | T | F | T | T | T |
| f | f | t | t | t | t | T |

The truth table shows it is a tautology as the expression is true for all values of p and q

* 1. (p→q)∧(q→r)→(p→r)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | Q | R | p→q | q→r | (p→q)∧(q→r) | p→r | (p→q)∧(q→r)→(p→r) |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| f | f | f | t | t | t | t | T |

This is a tautology as the expression evaluated to true for all values of p, q and r

* 1. (p→q)∧(p∧¬q)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| P | Q | p→q | ¬q | p∧¬q | (p→q)∧(p∧¬q) |
| T | T | T | F | F | F |
| T | F | F | T | T | F |
| F | T | T | F | F | F |
| f | f | t | t | f | F |

This is a contradiction as the truth table shows that for all values of p and q the expression is false

1. 1. (p∨p)∧p≡p

|  |  |  |
| --- | --- | --- |
| 1. | (p∨p)∧p |  |
| 2. | p∧p | Idempotent Law 1 |
| 3. | p | Idempotent Law 2 |

* 1. (p→(q∨r))≡p∧¬q→r

|  |  |  |
| --- | --- | --- |
| 1. | p∧¬q→r |  |
| 2. | ¬(p∧¬q)∨r | Conditional Identity 1 |
| 3. | ¬p∨¬¬q∨r | DeMorgans Law 2 |
| 4. | ¬p∨q∨r | Double Negation Law 3 |
| 5. | p→(q∨r) | Conditional Identity 4 |

* 1. p≡ ¬p→(p∧q)

|  |  |  |
| --- | --- | --- |
| 1. | ¬p→(p∧q) |  |
| 2. | ¬¬p∨(p∧q) | Conditional Identity 1 |
| 3. | p∨(p∧q) | Double Negation law 2 |
| 4. | P | Absorption Law 3 |

* 1. (¬p∧¬q)→r≡¬p→(q∨r)

|  |  |  |
| --- | --- | --- |
| 1. | (¬p∧¬q)→r |  |
| 2. | ¬(¬p∧¬q)∨r | Conditional Identity 1 |
| 3. | ¬¬p∨¬¬q∨r | DeMorgans Law 2 |
| 4. | ¬¬p∨q∨r | Double Negation Law 3 |
| 5. | ¬p→(q∨r) | Conditional Identity 4 |



|  |  |  |
| --- | --- | --- |
| 1 | p→(q∧r) | hypothesis |
| 2 | ¬p∨(q∧r) | Conditional Identity 1 |
| 3 | (¬p∨q)∧(¬p∨r) | Distributive law 2 |
| 4 | (p→q) ∧ (p→r) | Conditional Identity 3 |
| 5 | p→r | Hypothetical Syllogism 4 |
| 6 | ¬r | Hypothesis |
| 7 | ¬p | Modus Tollens 5,6 |



|  |  |  |
| --- | --- | --- |
| 1 | (¬p)∧q→¬r | Hypothesis |
| 2 | ¬(¬p∧q)∨¬r | Conditional Identity 1 |
| 3 | (¬¬p∨¬q)∨¬r | DeMorgans Law 2 |
| 4 | (p∨¬q)∨¬r | Double Negation Law 3 |
| 5 | ¬r∨(p∨¬q) | Associative law 4 |
| 6 | r | Hypothesis |
| 7 | ¬¬r | Double Negation Law 6 |
| 8 | (p∨¬q) | Disjunctive Syllogism 5, 7 |
| 9 | ¬q∨p | Associative Law, 8 |
| 10 | q | Hypothesis |
| 11 | ¬¬q | Double Negation 10 |
| 12 | p | Disjunctive Syllogism 9, 11 |



|  |  |  |
| --- | --- | --- |
| 1 | p∨q | Hypothesis |
| 2 | ¬p∨r | Hypothesis |
| 3 | q∨r | Resolution 1,2 |
| 4 | r∨q | Commutative Law 3 |
| 5 | ¬q | Hypothesis |
| 6 | r | Disjunctive Syllogism 3,4 |
| 7 | ¬r∨s | Hypothesis |
| 8 | S | Disjunctive Syllogism 5,6 |

1. 1. p⊕q ≡ q⊕p

|  |  |  |
| --- | --- | --- |
| 1 | p⊕q |  |
| 2 | (p∨q)∧¬(p∧q) | Definition 1 |
| 3 | (q∨p)∧¬(p∧q) | Commutative Law 2 |
| 4 | (q∨p)∧¬(q∧p) | Commutative Law 3 |
| 5 | q⊕p | Definition 4 |

* 1. p⊕p is a contradiction

|  |  |  |
| --- | --- | --- |
| 1 | p⊕p |  |
| 2 | (p∨p)∧¬(p∧p) | Definition 1 |
| 3 | p∧¬(p∧p) | Idempotent Law 2 |
| 4 | p∧¬p | Idempotent Law 3 |
| 5 | False | Complement Law 4 |

It will always evaluate to false as the expression simplifies to false

* 1. r∧(p⊕q) ≡ (r∧p)⊕(r∧q)

|  |  |  |
| --- | --- | --- |
| 1 | (r∧p)⊕(r∧q) |  |
| 2 | (r∧p)∨(r∧q))∧¬((r∧p)∧(r∧q)) | Definition 1 |
| 3 | (r∧(p∨q))∧ ¬((r∧p)∧(r∧q)) | Distributive Law 2 |
| 4 | (r∧(p∨q))∧(¬(r∧p)∨¬(r∧q)) | DeMorgans Law 3 |
| 5 | (r∧(p∨q))∧((¬r∨¬p)∨¬(r∧q)) | DeMorgans Law 4 |
| 6 | (r∧(p∨q))∧((¬r∨¬p)∨(¬r∨¬q)) | DeMorgans Law 5 |
| 7 | (r∧(p∨q))∧(¬r∨¬r∨¬p∨¬q) | Associative Law 6 |
| 8 | (r∧(p∨q))∧(¬(r∧r)∨¬p∨¬q) | DeMorgans Law 7 |
| 9 | (r∧(p∨q))∧(¬r∨(¬p∨¬q)) | Idempotent Law 8 |
| 10 | (r∧(p∨q)∧¬r)∧(r∧(p∨q)∧¬p)∨(r∧(p∨q)∧¬q) | Distributive Law 9 |
| 11 | (r∧¬r∧(p∨q))∧(r∧(p∨q)∧¬p)∨(r∧(p∨q)∧¬q) | Associative Law 10 |
| 12 | (F∧(p∨q))∨((r∧(p∨q)∧¬p)∨(r∧(p∨q)∧q)) | Negation Law 11 |
| 13 | F∨((r∧(p∨q)∧¬q)∨(r∧(p∨q)∧¬q)) | Domination Law 12 |
| 14 | (r∧(p∨q))∧(¬p∨¬q) | Distributive Law 13 |
| 15 | r∧(p∨q)∧¬(p∧q) | DeMorgans Law 14 |
| 16 | r∧(p⊕q) | Definition |

1. p: The student got an A on the final.

q: The student turned in all the homework.

r: The student is on academic probation

* 1. The student is not on academic probation and the student got an A on the final or turned in all the homework.

≡ ¬r∧(p∨q)

* 1. If the student got an A on the final, then the student is not on academic probation

≡ p→¬r

* 1. If the student is on academic probation, then the student did not get an A on the final

≡ r→¬p